

# METU - NCC

LINEAR ALGEBRA SHORT EXAM 1					
Code	: MAT 260	Last Name:			
Acad. Year	: 2014-2015	Name			
Semester	: SPRING	Student #			
Date	: 17.03.2014	Signature			
Time	: 12:30	3 QUESTIONS ON 2 PAGES TOTAL 10 POINTS			
Duration	: 50 min				
1. (3)	2. (3)	3. (4)			

1. (3pts) Let  $E = \{(1, 2, 3, -1), (1, 0, 2, 1), (1, 1, 1, 1)\}$ . Show that  $E$  is linearly independent.

$$\lambda(1, 2, 3, -1) + \mu(1, 0, 2, 1) + \theta(1, 1, 1, 1) = 0$$

$$\begin{cases} \lambda + \mu + \theta = 0 \\ 2\lambda + \theta = 0 \\ 3\lambda + 2\mu + \theta = 0 \\ -\lambda + \mu + \theta = 0 \end{cases} \Rightarrow \lambda = 0, \theta = 0, \mu = 0$$

2. (3pts) Let  $S = \{a, b, c\}$ ,  $V = \text{Fun}(S)$  and  $E = \{2x_a + 3x_b - x_c, 2x_a + x_b + 3x_c, 2x_a + 2x_b + x_c\}$ . Find a linearly independent subset of  $E$ .

$$(2, 3, -1), (2, 1, 3), (2, 2, 1)$$

$$\lambda(2, 3, -1) + \mu(2, 1, 3) + \theta(2, 2, 1) = 0$$

$$\begin{cases} 2\lambda + 2\mu + 2\theta = 0 \\ 3\lambda + \mu + 2\theta = 0 \\ -\lambda + 3\mu + \theta = 0 \end{cases} \Rightarrow \begin{cases} \lambda - \mu = 0 \Rightarrow \lambda = \mu \\ 2\mu + \theta = 0 \Rightarrow \theta = -2\mu \end{cases}$$

$$\mu(2, 3, -1) + \mu(2, 1, 3) - 2\mu(2, 2, 1) = 0$$

$$\mu = 1$$

$$(2, 3, -1) + (2, 1, 3) = 2(2, 2, 1)$$

$$U = \{2x_a + 3x_b - x_c, 2x_a + x_b + 3x_c\}$$

$$\lambda(2, 3, -1) + \mu(2, 1, 3) = 0$$

$$\begin{cases} 2\lambda + 2\mu = 0 \\ 3\lambda + \mu = 0 \\ -\lambda + 3\mu = 0 \end{cases} \Rightarrow \begin{matrix} 8\mu = 0 \\ \mu = 0 \end{matrix} \Rightarrow \lambda = 0.$$

$U$  is a lin. ind. subset of  $E$

3. (4pts) Let  $V = \mathcal{P}_3(\mathbb{R})$  be the vector space of all polynomials of their degrees at most 3, and let  $U = \{a_0 + a_1x + a_2x^2 + a_3x^3 \in V : a_0 + a_1 - 2a_2 - a_3 = 0\} \subseteq V$  be the subspace. Find a linearly independent subset  $E$  of vectors from  $U$  such that  $\text{Span}(E) = U$ .

$$P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$$

$$a_0 + a_1 - 2a_2 - a_3 = 0$$

$$x + y - 2z - 0 = 0.$$

$$(1, -1, 0, 0), (2, 0, 1, 0), (1, 0, 0, 1)$$

$$(a_0, a_1, a_2, a_3) \in U$$

$$(a_0, a_1, a_2, a_3) = \lambda(1, -1, 0, 0) + \mu(2, 0, 1, 0) + \theta(1, 0, 0, 1)$$

$$a_0 = \lambda + 2\mu + \theta$$

$$\lambda = -a_1$$

$$\mu = a_2$$

$$\theta = a_3$$

$$(a_0, a_1, a_2, a_3) = -a_1(1, -1, 0, 0) + a_2(2, 0, 1, 0) + a_3(1, 0, 0, 1)$$

$$E = \{1-x, 2+x^2, 1+x^3\}$$